Beehive Geometry

Investigation into Area Efficiency of *n*-sided Polygons of Equal Perimeter

Lesson Summary

In this lesson, students will use area calculations to investigate the relationship between the number of sides in a regular polygon and the area given equal perimeter.

Students will be given the scenario of hexagonal beehives for context and be introduced to tessellations. They will exercise their reasoning skills through class discussions and engage in discovery-based learning by using virtual manipulatives. They will apply their knowledge in an investigation by practicing calculating the areas of a triangle, square, and hexagon.

The accompanying materials for this lesson include the lesson plan with hyperlinks in the Materials section and the associated worksheet.

Inquiry Question

Why do honey bees build with hexagons, and how can we investigate this using geometry?

Estimated Time

- 50 min

Required Materials

- Laptops/devices for students
- Projector
- Internet access for online tools
- Worksheet

Vocabulary

Tessellation

- An arrangement of shapes that fit together with no gaps and no overlaps.

Lesson Details

Grade Level

- Grade 9

Subject Area Connections

Subject Areas

- Math, Grade 9 De-streamed MTH1W

Curriculum Expectations

MTH1W:

- E1.1 research a geometric concept or a measurement system to tell a story about its development and use in a specific culture or community, and describe its relevance in connection to careers and to other disciplines
- E1.4 show how changing one or more dimensions of a two-dimensional shape and a three-dimensional object affects perimeter/circumference, area, surface area, and volume, using technology when appropriate

Duration

- 50min + 25min for questions and homework time, i.e. 75min (one period)

Learning Objectives

Students will:

- Be introduced to the concept of tessellations and visualize them using virtual manipulatives
- Investigate the relationship between area and number of sides in a regular polygon when perimeter stays the same
- Practice calculating and comparing areas of regular polygons
- Relate this mathematical concept with the real-life application of honey bees

Background Knowledge

Suggested prior knowledge:

- Area of a triangle and area of a rectangle
- Names of polygons
- Composite shapes and how to calculate the area

Summary of Activities

1. Introduction to why honey bees make their hives with hexagons through placing students in the scenario as a bee and using logical reasoning to infer why hexagons may be used.
2. Introduction to tessellations using Polypad, finding out which regular polygons tessellate with themselves.
3. Worksheet where students calculate the area of these tessellating polygons if the perimeter remains the same.
4. Discussion of the relationship between the number of sides in a regular polygon and the area, given a constant perimeter.
5. Consolidation with a short BBC video about honey bees and their use of hexagons.

Considerations for a Safe Learning Environment

- Some students may not be aware of what honeycombs look like, in which case an explanation, drawing, or image at the beginning of class may be beneficial. However, be careful of student phobias (see below).
- Some students may have trypophobia, which can be triggered by a honeycomb pattern, or a phobia of bees/insects, in which case the teacher should be careful about including images of these things.

Assessment

- Assessment for learning (formative assessment), mostly through circulation and using plenty of discussion periods with online response options.

Accommodations

- Multiple means of expression during discussion sections with an out loud and online response option (teachers can set up a Mentimeter or Padlet or something similar). This also results in increased processing time.
- Group work allows students to collaborate, and allows students without a device to still be able to use the virtual manipulatives when grouped with a student that does have a device.

Lesson Plan

Set Up

Materials	Preparation
Students - Laptops/Chromebooks, if available (communicate this ahead of time) Teachers: *Physical:* - Computer + projector - Worksheet *Digital:* - Polypad, polygon tool - BBC video - Mentimeter/Padlet/other preferred means of response collection	Classroom Setup: - If needed, book a Chromecart/laptop cart for the class - Students should ideally be in pairs or table groups, but single desk seating works too. Digital Setup: *Open the following ahead of time:* - Polypad *Be prepared to:* - Show the correct tool to use on Polypad - Demonstrate tessellations on Polypad to debrief the activity

Instructive Outline

Total time: 50min + 25min for questions and homework.

Instruction	Notes and Assessment
Intro/Task Launch/Minds On (~10 min)	
Story and scenario (5 min): - We know that honey bees use beeswax to build a bunch of little hexagons, which makes up their hive. We're going to consider the mathematical reasons why this happens. - "Imagine you are a honey bee who lives in a hive. You need to choose one shape for a "cell", which will repeat itself to make up the whole hive. You'll make it out of beeswax that bees produce and you store honey and live in the cells."	Discussion contributions can be received verbally or with Mentimeter depending on student comfort, offering multiple means of expression. Assessment for

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- Discussion: “As a honey bee architect, what features would you want this cell to have?”
  - Make sure to get space/area/perimetre efficiency
    - Which means there shouldn’t be gaps between cells. The shape needs to leave no gaps and have no overlaps when tiling the surface. This is called a **tessellation**.

Polypad tessellations (5 min):

- Investigate: What shapes from our regular polygons tessellate?
  - Go to “Polygon” under “Geometry” on the Polypad site.
  - Experiment with each polygon and see if you can create a tessellation with just one shape. Which shapes can you do this with?
  - Answer: Triangle, square, hexagon.
- Discussion: If they all tessellate, then how do we decide which shape is the best to use for our beehive?
  - Answer: Area efficiency, i.e. biggest area per cell with the same amount of perimeter (beeswax)

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learning:

- Circulate to see what students are getting on Polypad and to make sure they are using the correct Polypad tool.

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<u>Action</u> (~30 min)

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Area investigation (15 min):

- Hand out worksheet. Work in groups of 1-3 (encourage discussion and collaboration).
  - Consider a triangle, square, and hexagon of equal perimeter. Calculate the area of each one.
    - Hint, as a reminder of composite shapes: You don’t know the formula for the area of a hexagon, but you might be able to split it up into smaller shapes that you do know

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Assessment for learning:

- Circulate, give hints and answer questions as needed. If there is a wide misconception, clarify it with the whole class.

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formulas for. - Based on this investigation, which shape would you want to use for your beehive and why? Share answers on Mentimeter (or other similar program, like Padlet). - Debrief answers, reveal the correct answer is hexagon. - What trends do you notice? Share on Mentimeter/Padlet/etc. - Debrief the answers and correct any misconceptions. Conclude that for the same perimeter, more sides means more area. - Discussion: If that's the case, then which shape do you think has the maximum area efficiency for perimeter? → Circle - Note that depending on what these shapes are being used for, what's considered the "best" shape for the job might differ. In a general sense, with no other requirements, a circle is best for area efficiency. But in the case of our bees, we need to tessellate them and circles would leave gaps. So instead we use hexagons. - Discussion: In what other applications or jobs do you imagine that this idea of space efficiency is important?	Note that students without a device can work with students that do have a device. If needed (such as if students do not have a formula sheet), write the formulas for area of a triangle and area of a square on the board.
<u>Consolidation</u> (~5 min)	
BBC video (5 min): - Show the video which summarizes the math behind bees using hexagons. Homework time: - Any extra time can be used for students to complete any homework or ask questions.	

Extensions

- If students know right angle trigonometry (such as in Grade 10 or 11), then area calculations can be done using trigonometry for the hexagon (and pentagon, octagon, etc.)

Potential Future Lessons

- Optimization (using functions) for Grades 10 or 11